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AN INVESTIGATION OF BROADSCALE VERTICAL MOTIONS AND PRECIPITATION IN AN EASTERN NORTH AMERICAN STORM

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ABSTRACT

The precipitation associated with the intense storm of February 25–27, 1961, which affected eastern North America, is investigated. Vertical motion patterns are computed for successive days of the storm using a method suggested by Bushby, and based on the Sutcliffe development theory. These patterns are used, together with different rate of precipitation formulas, to calculate precipitation amounts. Comparisons with actual amounts are made. A method of quantitative precipitation forecasting is suggested.

1. INTRODUCTION

The storm of February 25–27, 1961 will long be remembered in southern Quebec, not so much for the precipitation amounts which occurred, but because the precipitation remained in the form of freezing rain throughout the period. Heavy deposits of ice formed on trees, and hydro and communication lines. Accompanying strong northeast winds snapped thousands of hydro poles, especially in areas where lines were oriented at large angles to the wind direction. Most of the area was without electric power for three days, and some communities did not return to normal for more than a week.

Major deepening of the Low took place during the period 0600 GMT February 25 (1000 mb.) to 1800 GMT February 25 (978 mb.) During this period the cyclonic vorticity maximum rounded the 500-mb. trough and moved northeastward. The strong vorticity advection contributed considerably to the deepening of the surface Low in agreement with the development formula of Petterssen [10]. Another factor which no doubt helped the explosive deepening was the low-level instability of the inflowing maritime tropical air over the Southeastern United States. An examination of the raobs on the morning of the 25th in this area, and especially of the 1200 GMT Greensboro raob, revealed the marked instabil-

ity present in this air mass. In general, the buoyancy term contribution toward deepening of the system was becoming well established by early on the 25th.

2. THE VERTICAL VELOCITIES IN THE SYSTEM

Since the rates of condensation and precipitation are largely determined by the adiabatic cooling resulting from upward motion, it would appear reasonable that in a system which produces large areas of heavy precipitation, there should be an associated large broadscale upward motion of air.

The vorticity equation is used as a basis for computing vertical motion. In its simplified form, neglecting vertical advection of vorticity, and the so-called tilting-twisting term, it becomes:

$$\left[\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right] (\zeta + f) = -f \nabla \cdot \mathbf{V} \quad (2.1)$$

An expression has been derived by Sutcliffe [14] for the tendency of the thickness h between pressure levels p_0 and p_1 :

$$\frac{\partial h}{\partial t} = -\mathbf{V}_m \cdot \nabla h + \frac{R}{g} \Gamma \bar{\omega} \log \frac{p_0}{p_1} \quad (2.2)$$

In the above the stability $\Gamma [= (dT/dz) - (\partial T/\partial z)]$ is taken constant for the layer, and $\bar{\omega}$ is the mean value of ω ($= dp/dt$) for the layer; T is temperature, R the gas constant, and g the acceleration of gravity; and subscript m designates the mean value for the layer.

From equations (2.1) and (2.2) and using the geostrophic assumption that $\zeta = (g/f)\nabla^2 z$ it follows that:

$$\mathbf{V}_h \cdot \nabla(\zeta_0 + \zeta_1) + \frac{R}{f} \nabla^2(\Gamma \bar{\omega}) \log \frac{p_0}{p_1} = f \left[\frac{\partial \omega}{\partial p} \right]_0 \quad (2.3)$$

where \mathbf{V}_h is the thermal wind.

If the assumption is made that $\nabla \cdot \mathbf{V}$ is a linear function of pressure, so $\nabla \cdot \mathbf{V} = a(p - p_2)$ then, since $\partial \omega / \partial p = -\nabla \cdot \mathbf{V}$ from the equation of continuity, ω will have a parabolic form with respect to pressure, that is:

$$\omega = \frac{a}{2} (p_0 - p)(p_0 - 2p_2 + p)$$

Equation (2.3) is a Helmholtz equation of the form:

$$\nabla^2(\Gamma \bar{\omega}) + A \bar{\omega} + B \mathbf{V}_h \cdot \nabla(\zeta_0 + \zeta_1) = 0 \quad (2.4)$$

where A and B are dependent upon p_0 , p_1 , g , R , mean temperature and the Coriolis parameter. Since horizontal variations of mean temperature and Coriolis parameter are small compared with the absolute values, A and B can be treated as constants.

Equation (2.4) can be solved readily on a computer. Bushby [3] found experimentally that if there is no damping due to vertical stability [i.e., $\nabla^2(\Gamma \bar{\omega}) \equiv 0$], the effect is to multiply $\bar{\omega}$ by 4, but not to change the pattern appreciably. This multiplier was adjusted to 2.8 in order to produce consistency between the observed non-advective component of the thickness tendency, and the same component as given by the second term in equation (2.2).

If we restrict the method to those situations where the 1000–500-mb. column of air is not in neutral or unstable equilibrium (Γ negative), then the vertical velocity can be written in the form:

$$\bar{\omega} = -\frac{1}{2.8} \frac{B}{A} \mathbf{V}_h \cdot \nabla(\zeta_0 + \zeta_1) \quad (2.5)$$

The vorticity at levels p_0 and p_1 can be found from the equation:

$$\zeta_0 + \zeta_1 = \frac{g}{f} \nabla^2 (z_0 + z_1) = \frac{4g}{fd^2} [\overline{(z_0 + z_1)} - (z_0 + z_1)]$$

where the bar indicates a space-mean value.

If the pressure levels are taken at 1000 mb. and 500 mb., and if the grid-distance d is taken as 6 latitude degrees at 45° N., the mean vertical velocity may be written in the simplified form:

$$\bar{\omega} = 1.39 \times 10^{-3} \mathbf{V}_h \cdot \nabla \zeta_{10+5}^* \text{ cm. sec.}^{-1} \quad (2.6)$$

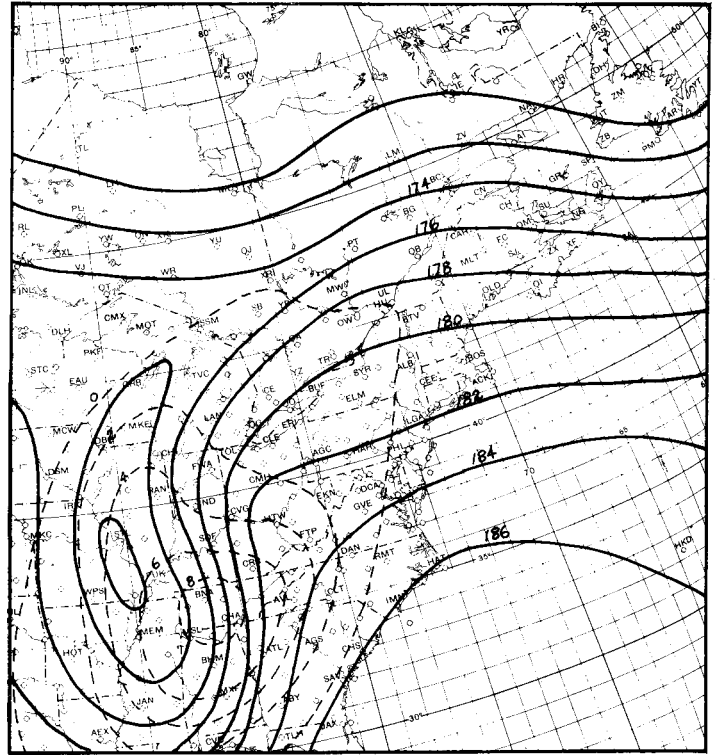


FIGURE 1.—February 25, 1961, 1200 GMT. The solid line is 1000–500-mb. thickness in hundreds of feet and the dashed line is $(z_{10} + z_5) - (z_{10} + z_5)$.

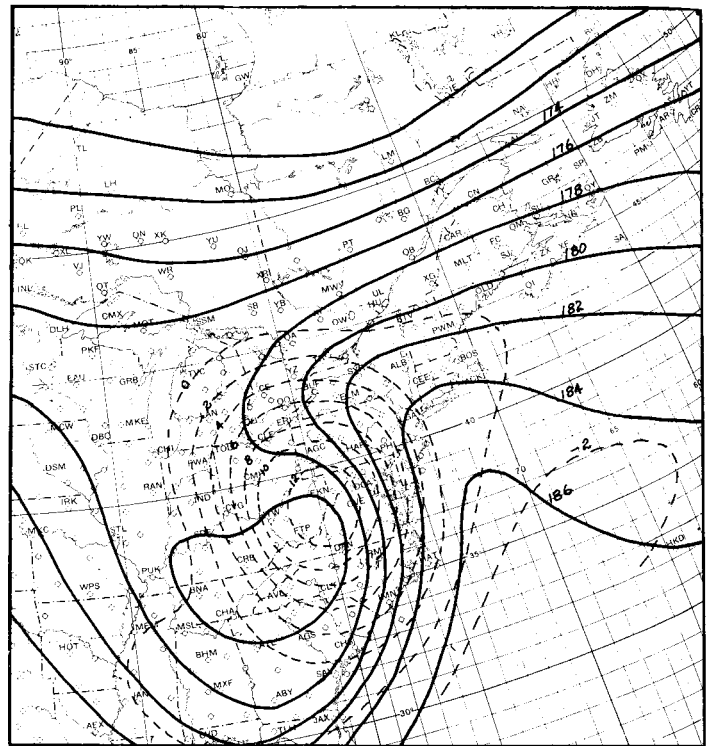


FIGURE 2.—February 26, 1961, 0000 GMT. The solid line is 1000–500-mb. thickness in hundreds of feet and the dashed line is $(z_{10} + z_5) - (z_{10} + z_5)$.

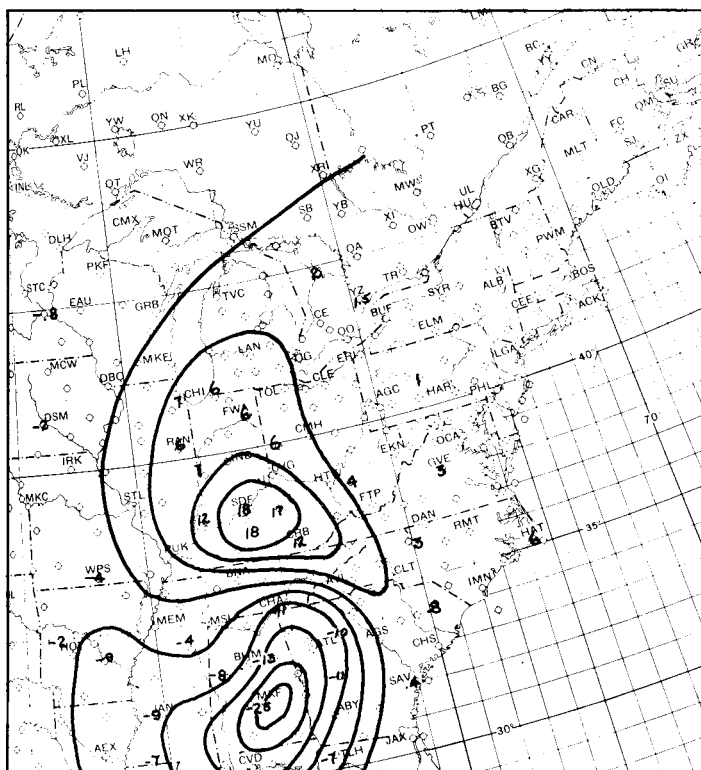


FIGURE 3.—February 25, 1961, 1200 GMT, mean vertical velocity \bar{w} (cm. sec.⁻¹).

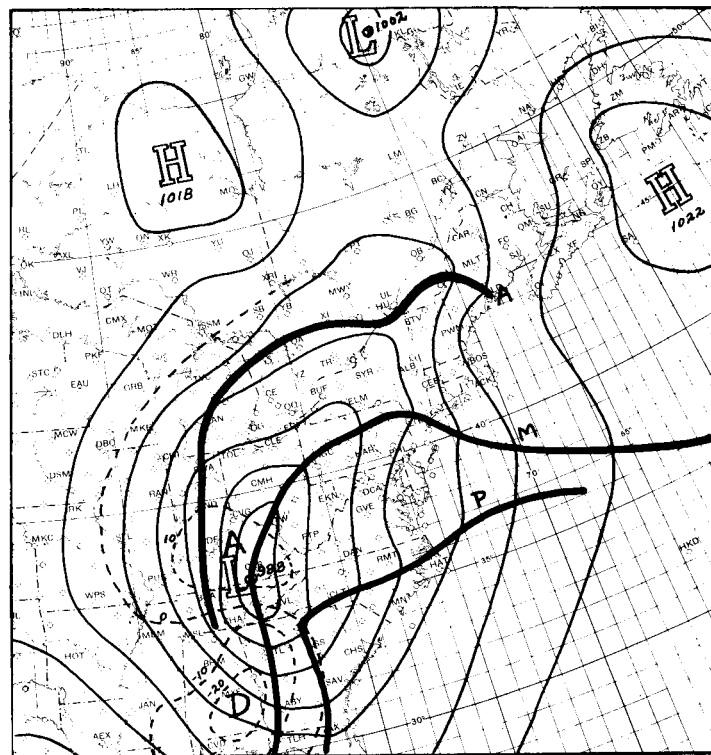


FIGURE 5.—February 25, 1961, 1200 GMT, surface chart. The dashed lines are vertical motion (cm. sec.⁻¹).

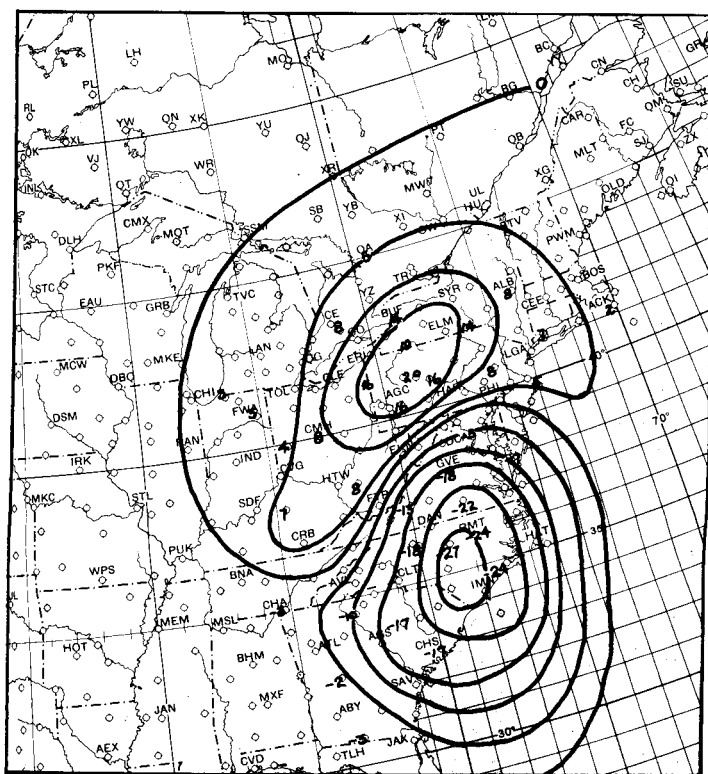


FIGURE 4.—February 26, 1961, 0000 GMT, mean vertical velocity \bar{w} (cm. sec.⁻¹).

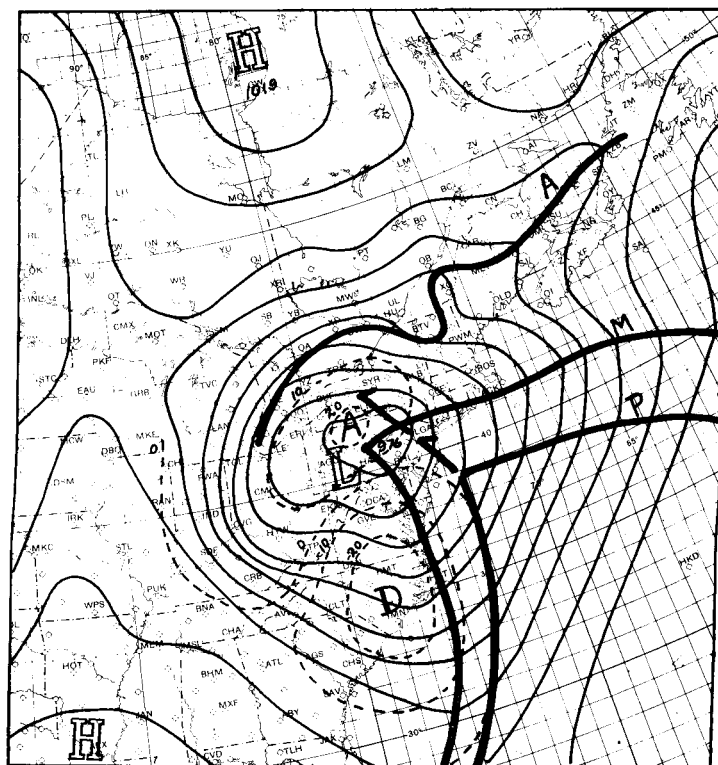


FIGURE 6.—February 26, 1961, 0000 GMT, surface chart. The dashed lines are vertical motion (cm. sec.⁻¹).

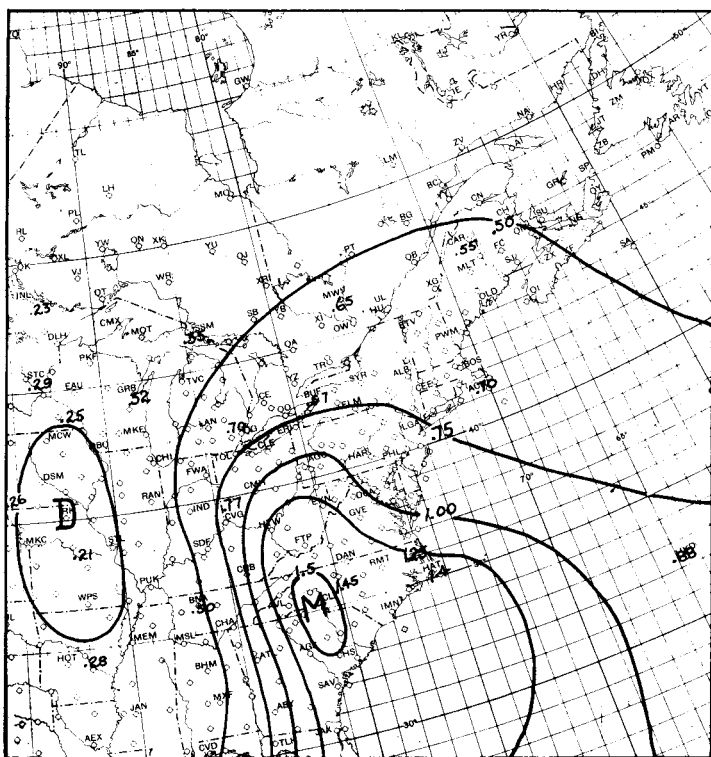


FIGURE 7.—February 25, 1961, 1200 GMT, total precipitable water (in.).

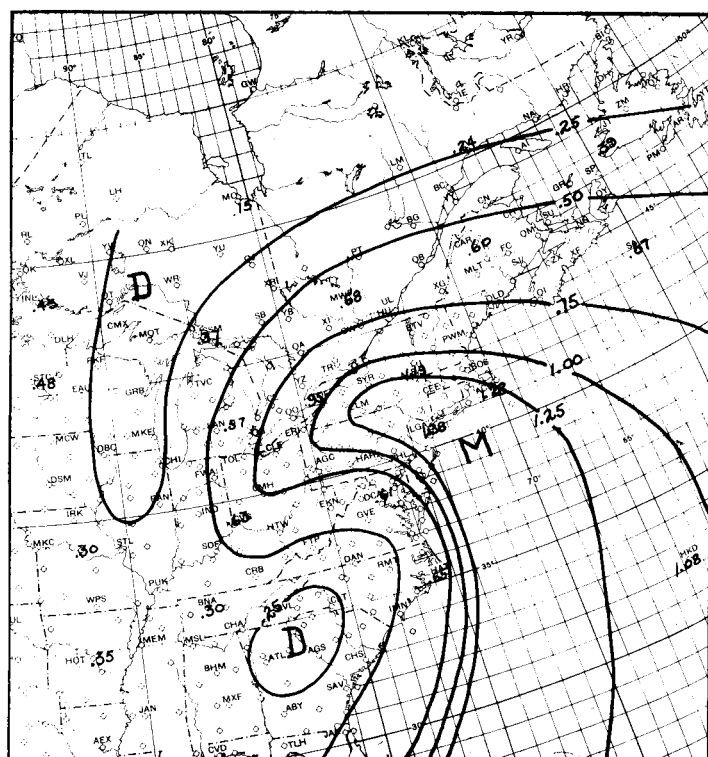


FIGURE 8.—February 26, 1961, 0000 GMT, total precipitable water (in.).

where w is the component of velocity in the z -direction and where ζ_{10+5}^* is the pseudo-vorticity of the $z_{10}+z_5$ field so that $\nabla \zeta_{10+5}^*$ represents the gradient of $[(z_{10}+z_5) - (z_{10}+z_5)]$ in the direction of \mathbf{V}_h . Thus \bar{w} is inversely proportional to the area of the quadrilaterals formed by the thickness lines and the $(z_{10}+z_5) - (z_{10}+z_5)$ lines. This result has been used by Grundy [8] in examining the relative magnitudes of the terms in Petterssen's development equation.

The total thickness (1000–500 mb.) and the pseudo-vorticity field of the $z_{10}+z_5$ surface for 1200 GMT February 25 and 0000 GMT February 26 are shown in figures 1 and 2.

The mean vertical velocities for the layer, calculated by equation (2.6), are illustrated in figures 3 and 4. Mean ascent values of up to 20 cm. sec.⁻¹ were computed. These values are somewhat greater than found by Sawyer [12], Graham [7], and Fleagle and Izumi [5], but are of an order of magnitude similar to that calculated by Sanders [11] and Brown and Sanders [2]. It must be noted that this was a particularly intense storm, and also that the vertical velocities of over 15 cm. sec.⁻¹ are restricted to rather small areas.

Figures 5 and 6 show the relation of the computed vertical motion pattern to the surface frontal and isobaric pattern.

3. THE MOISTURE CONTENT OF THE AIR MASSES

The total precipitable water amounts are calculated by a method described by Solot [13]. This makes use of a continuous dew point curve throughout the tephigram plot. Alternate methods for making a computation are described by Creswick [4] and by Peterson [9]. Since much of the moisture in this case is concentrated below the 850-mb. level but not evident from surface dew points, due to shallow surface inversions, the more precise computations are considered necessary. Resulting precipitable water patterns are shown in figures 7 and 8. These illustrate the moisture present but are not used in the actual calculations.

4. PRECIPITATION AMOUNTS

The question arises: Are the computed vertical motions reasonable values, and are they consistent with the precipitation amounts which occurred in the storm? Consequently, if the surface and 500-mb. patterns, and the moisture configuration were forecast, could a quantitative precipitation forecast be made?

The actual precipitation amounts during 24-hr. periods, ending at 0000 GMT February 26 and 0000 GMT February 27 are shown in figure 9. It illustrates that an area of 2–3 in. of precipitation moved with the track of the storm.

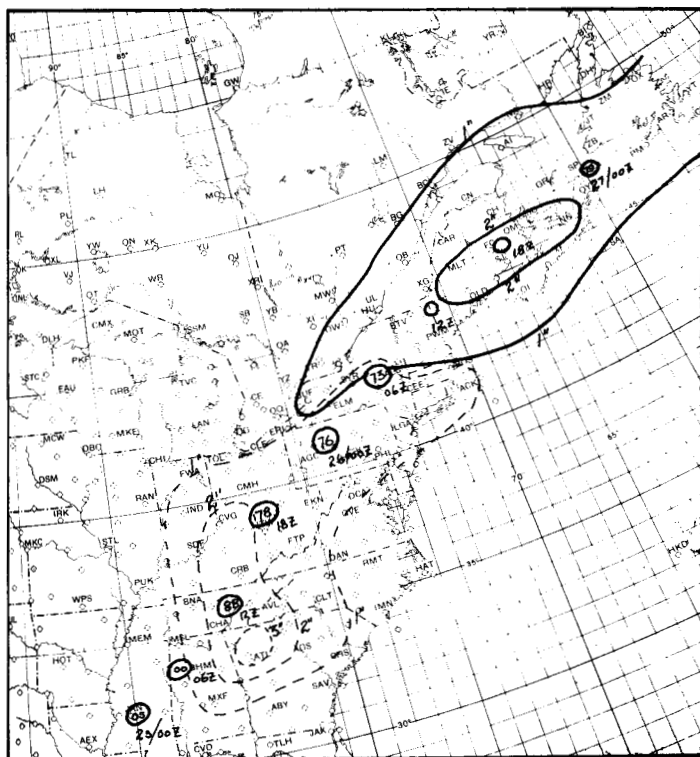


FIGURE 9.—24-hr. precipitation (in.) ending 1200 GMT, February 26, 1961, (dashed line), and ending 1200 GMT, February 27, 1961 (solid line). Low positions and central pressure circled; date and time at each position indicated by number group at lower right side of circle.

On February 25 amounts of somewhat greater than 3 in. occurred in eastern Tennessee and northern Georgia. It would appear likely that the release of potential instability in an upslope effect in this region would have contributed considerably to the rainfall amounts.

On February 26 and 27 however, as the Low tracked northeastward, north of the main Appalachian Range, the broadscale ascent of the air mass should essentially account for the rain which fell.

The precipitation rate P (in. hr.⁻¹) has been related to the mean vertical velocity \bar{w} (cm. sec.⁻¹) by Bannon [1] by the formula:

$$P = \frac{.039\bar{w}}{F} \quad (4.1)$$

where F is a function related to the wet-bulb potential temperature θ_w as in table 1. The assumptions made are that all precipitation is due to the general lifting of the

TABLE 1.—The relationship between the function F and the wet-bulb potential temperature θ_w .

θ_w	15° C.	10° C.	5° C.
F	2.9	3.8	5.5

TABLE 2.—The total precipitation amount calculated for an area in the track of the Low.

\bar{w}	Lateral extent of \bar{w} (mi.)	Time t exposed to \bar{w} (hr.)	Precipitation rate P (in. hr. ⁻¹)	Precipitation in time t (in.)
15 cm. sec. ⁻¹	150	$\frac{150}{35} = 4.3$	$\frac{0.039 \times 15}{2.9} = 0.20$	0.86
10 cm. sec. ⁻¹	350	$10 - 4.3 = 5.7$	$\frac{0.039 \times 10}{2.9} = 0.13$	0.74
5 cm. sec. ⁻¹	600	$17 - 10 = 7$	$\frac{0.039 \times 5}{2.9} = 0.07$	0.47
Total				2.07 in.

air mass, and that the evaporation of falling rain is negligible. The wet-bulb potential temperature is read directly from the tephigrams where it is plotted as standard procedure in the Central Analysis Office.

For an area in the track of the Low and thus of the maximum ascent region on February 25–27, taking into account that the system moved at 35 kt., the precipitation amount is calculated in table 2.

An alternate method of computing precipitation rate is given by Godson [6], and involves the concept of entropy. If the change of entropy of dry air ascending pseudo-adiabatically, is symbolized by ΔS_a (I.T. cal. gm.⁻¹ °K.⁻¹), and if ρ is the density (gm. cm.⁻³), then the rate of precipitation may be written:

$$R_p \text{ (in. per 6 hr.)} = 38 \cdot 10^2 \sum \rho w (\Delta S_a) \theta_w \quad (4.2)$$

The summation is done for 100-mb. layers, and the value of w for the middle of each layer is used, as given by the parabolic relationship between vertical velocity and pressure. The layer contributions to R_p are given in table 3.

An area in the track of the storm is exposed to varying vertical motions, which however, correspond to $\bar{w} = 10$ cm. sec.⁻¹ for 15 hr. During the same period it is exposed to an air mass of θ_w 16 above the 800-mb. level and of

TABLE 3.—Layer contributions to the precipitation rate, and resulting total precipitation rate. (According to Godson [6].)

Rates of precipitation for saturated pseudo-adiabatic lapse rate with a mean vertical velocity of 10 cm. sec. ⁻¹				
Layer (mb.)	ρw for layer (10 ⁻³ gm. cm. ⁻² sec. ⁻¹)	Layer contribution to R_p (in. per 6 hr.)		
		$\theta_w = 16^\circ \text{C.}$	$\theta_w = 10^\circ \text{C.}$	$\theta_w = 6^\circ \text{C.}$
1000–900	2.99	0.04	0.04	0.03
900–800	7.87	0.12	0.10	0.08
800–700	10.60	0.17	0.13	0.11
700–600	12.60	0.19	0.15	0.10
600–500	12.60	0.19	0.11	0.07
500–400	10.60	0.13	0.07	0.03
400–300	7.87	0.05	0.01	
300–200	2.99			
Total		0.89	0.61	0.42

θ_w 10 below that level. Thus a mean rate of precipitation is given by:

$$R_p = 0.87 \text{ in. per 6 hr.}$$

Hence the total precipitation during the storm:

$$P = \frac{15}{6} (0.87) = 2.18 \text{ in.}$$

It should be noted from table 3, that the air mass below the 800-mb. level is relatively unimportant in the precipitation computation, as the vertical motion remains small near the surface. The main contribution comes from the layer 800–500 mb.

5. CONCLUSIONS

The method described provides a fairly convenient objective means of computing vertical motions by hand. However, considerable care must be taken with the gridding process. The greatest time consumed is in calculating the term $\mathbf{V}_h \cdot \nabla \zeta_{10+5}^*$ at sufficient locations to delineate the complete pattern.

Precipitation amounts, computed for the location in the track of the region of maximum ascent, result in values which give an encouraging degree of correspondence with the amounts that fell.

Since after 1200 GMT February 25 the raobs from stations near the storm center indicated that the air masses were no longer unstable, the use of Bushby's abbreviated formula (2.5) appears justified. The effect of static stability (Laplacian term = 0) is likely to have resulted in considerably greater precipitation amounts, presumably up to four times those calculated.

The two methods used for computing precipitation resulted in almost identical values, Godson's method producing a slightly larger amount than that obtained from the Bannon formula.

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